CALCULATION OF SURFACE ORIENTATIONS OF TRANSPARENT OBJECTS BY LITTLE ROTATION USING POLARIZATION
微小回転と偏光解析に基づく透明物体の表面形状計算

by

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ABSTRACT

This paper introduces a method for obtaining surface orientations of transparent objects using polarization in highlight. We took a picture of the transparent object with a CCD camera with a polarizer in front of it, and through measuring the light intensity of the highlight of the transparent object, we calculate the value of polarization of each pixel. We calculate the degree of angle from polarization but for the nature of the equation, two angles will be produced. We propose the algorithm, which chooses the correct angle by comparing the two data of polarization by rotating an object very little. This paper shows the algorithm and reports its correctness through simulation results.

論文要旨

透明物体の表面形状を計測する手法として物体のハイライトの偏光解析を行う方法がある。透明物体のハイライト部分を、偏光板を通した CCD カメラで計測することによりカメラで撮影した画像の各ピクセルでの偏光度がわかる。偏光度から角度を求めるためにある式にあてはめて計算すると角度の値が２つでてくる。そこで、我々は物体を微小回転させ、２つの偏光度のデータを比較しながら２つの角度のうち正しい角度を選ぶアルゴリズムを開発した。この論文ではこのアルゴリズムの紹介をし、またその有用性をシミュレーションを通じて論じている。

Acknowledgments

We appreciate M. Saito's help and her work, which is the base of this work. We thank Prof. Ikeuchi and Y. Sato for useful advises and discussions.

Contents

1 Introduction

There is no easy non-contact method for measuring surface orientations of transparent object. Saito, Sato, Ikeuchi, and Kashiwagi[5] proposed to use the polarization in surface reflection components for determining shapes of transparent objects. This is a grateful contribution to measuring the surface orientations of transparent objects by easy non-contact method. The idea of measuring the polarization of the highlight of the object made a transparent object to be an easily measurable thing. Method are easy, we just take a picture with a CCD camera with a polarizer in front of it. While rotating the polarizer, we measure the light intensity of each pixel of each rotation of polarizer. We can calculate the degree of polarization from a certain equation with using the degree of the minimum light intensity and the maximum light intensity through the rotation. But unfortunately, since the candidates of the correct angle produced by this experiment are four angles, we should choose the correct angle to determine the surface shape.

We propose the method of determining the correct angle of the four angles calculated from polarization and the light intensity. We rotated the object to solve this problem, for example, in section 4.1 we show the simulation with the object rotated 1 degree from y-axis to z-axis. Since we should measure the both non-rotated object and rotated object, this method produces two matrixes. By comparing these two matrixes, we choose the correct angle automatically.

In Section 2, we review the reflection mechanism and shows how we actually set up an experimental apparatus. Section 3 describes the algorithm used to determine surface orientations. The algorithm complements the Saito's method. Section 4 describes the result of the experiment on simulation. Section 5 concludes the paper.

2 Basic Theory

2.1 Theory of Reflection and Polarization Light

The fact that material surface is not perfectly smooth causes four patterns of light reflected from material:

1. Light waves that specularly reflect off a planar interface with the surface.
2. Light waves that go through at least two multiple specular reflections amongst multiple microfacets.
3. Light waves that penetrate the material surface, internally multiply refract, and then refract back out into air.
4. Light waves that diffract from interfaces with surface detail the same size or smaller than the wavelength of the incident light wave.

The phenomenon 1 will be called the specular component of reflection. The phenomenon 2-4 will be called diffuse component. We call in another words the light caused by both phenomenon 1 and 2 as the highlight. The reflected light from material surface are the combination of these lights, but the light caused by phenomenon 3 and 4 are very little and can be ignored. The light caused by phenomenon 2 is also smaller than the light caused by phenomenon 1. The light caused by phenomenon 1 reflects as the same angle as incidence angle, so, by measuring the highlight, we can obtain the reflected angle. Figure 1 shows these four phenomena.

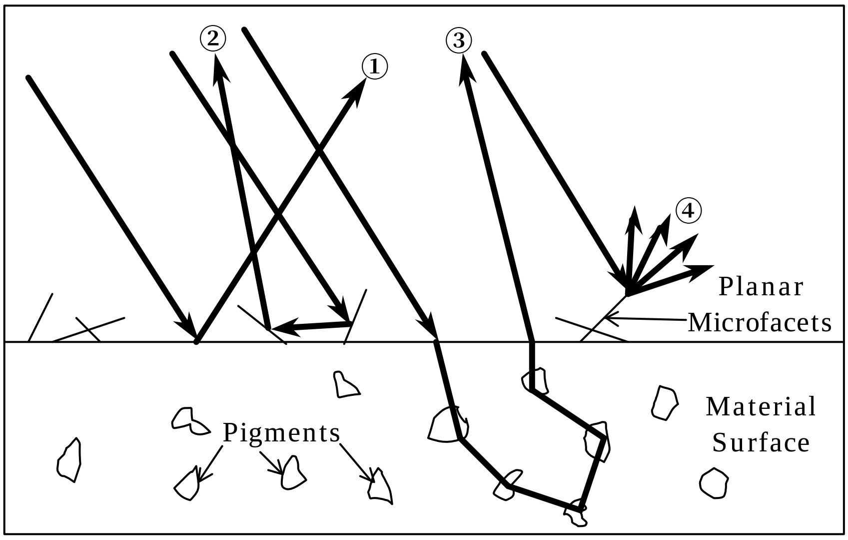


Figure 1: Reflection of light from a material surface

Consider the interface surface of medium 1 and 2, each refractive index is n_1 and n_2, is located in x-y plane as you see in Figure 2. In this case, the part of light refracts and transmits through the medium 2 and the other part of light reflects at the interface surface.

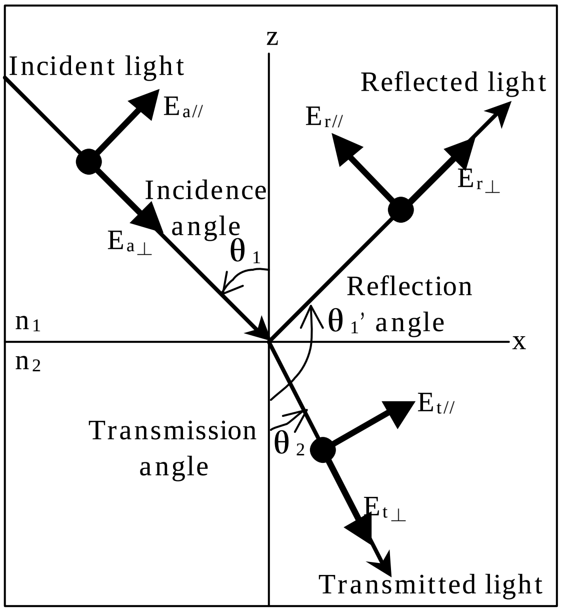


Figure 2: The Fresnel reflection

The electric field vector of incident, reflected, and transmitted light, are expressed as E_a, E_r, and E_t respectively, and parallel or perpendicular to the x-z plane are expressed by subscript $\|$ or $\perp$ respectively. The angle of incident, reflected, and transmitted light are expressed as q_1, q_1', q_2. Each electric field vector which is parallel to x-z plane are represented as follows:

$$
\begin{aligned}
E_{a\|} &= A_{\|} \exp\left[i\{wt - k_1\left(x \sin q_1 + z \cos q_1\right)\}\right] \\
E_{r\|} &= R_{\|} \exp\left[i\{wt - k_1\left(x \sin q_1' - z \cos q_1'\right)\}\right] \\
E_{t\|} &= T_{\|} \exp\left[i\{wt - k_2\left(x \sin q_2 + z \cos q_2\right)\}\right]
\end{aligned}
\tag{1}
$$

$A_{\|}$, $R_{\|}$, and $T_{\|}$ represents amplitude, w is $2p/l$, where l represents wavelength, and k_1 and k_2 represents the wave number in the medium 1 and 2.

The relationship between q_1 and q_1' is expressed as $q_1 = p - q_1'$, so the incident light and reflected light are symmetric at z-axis. And using this result, the following Snell's law will be given:

$$n_1 \sin q_1 = n_2 \sin q_2 \tag{2}$$

The boundary condition of the Maxwell equation requires that components of electric and magnetic field on boundary plane must be continuous at the plane. Thus, the linear combination of amplitude of the incident light and reflected light must equal to the amplitude of transmitted light with x and y direction. This is expressed as:

$$E_{aj} + E_{rj} = E_{tj}, \quad H_{aj} + H_{rj} = H_{tj} \quad (j = x, y) \tag{3}$$

In regarding to the Snell's law and the previous equations, we get:

$$r_{\parallel} = \frac{E_{r\parallel}}{E_{a\parallel}} = \frac{\tan(q_1 - q_2)}{\tan(q_1 + q_2)}$$
$$r_{\perp} = \frac{E_{r\perp}}{E_{a\perp}} = -\frac{\sin(q_1 - q_2)}{\sin(q_1 + q_2)} \tag{4}$$

$r_{\parallel}$ and $r_{\perp}$ is the reflectance of light amplitude, each is for parallel and perpendicular component. These equations (4) are called Fresnel's formula.

Light intensity is expressed as:

$$I = \frac{nE^2}{2\sqrt{m_0}} \tag{5}$$

Which n is refractive index of any medium. m_0 is permeability vacuum.

Using these equations shown above, intensity reflectance of parallel and perpendicular component is expressed as:

$$F_{\parallel} = \frac{\tan^2(q_1 - q_2)}{\tan^2(q_1 + q_2)}$$
$$F\perp = \frac{\sin^2(q_1 - q_2)}{\sin^2(q_1 + q_2)} \tag{6}$$

There is an angle, which makes $r_{\parallel}$ to 0. This incidence angle is called Brewster angle q_b. From $q_1 = p - q_1'$ and Snell's law, we get the following equation:

$$q_b = \arctan(n_2 / n_1) \tag{7}$$

When the incidence angle is equal to Brewster angle, reflectance light will be the linear polarized light of perpendicular component since the all parallel component will be transmitted.

2.2 Theory of Measurement

Natural light is an unpolarized light. Unpolarized light has equal magnitude polarization components in all directions. When the light reflected from the object which was initially a natural light source go through the polarizer, it oscillates sinusoidally as a function of polarizer angular orientation between a maximum light intensity $I_{\max}$ and a minimum light intensity $I_{\min}$. A measure of the proportion of how much initially unpolarized light becomes linearly polarized on reflection is given by

$$r = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}} \tag{8}$$

We call this as a degree of polarization r, which varies between 0 and 1 inclusive and represents the proportion of the magnitude of reflected light that is linearly polarized, relative to the total magnitude of reflected light. At $r = 0$, reflected light is unpolarized. At $r = 1$, reflected light is completely linearly polarized as predicted for pure specular reflection at the Brewster angle for a dielectric surface.

The geometry of our measurement system is shown in Figure 4. We define the plane of incidence as the one that includes the direction of a light source, a viewer, and a surface normal. In transparent objects, we can assume that the reflection angle is equal to the incidence angle, because we are measuring the highlight. We can obtain surface normal orientations by using orientation of the plane of incidence and reflection angle at each point of the object surface. We denote the orientation of the plane with f measured around the viewer's line of sight, and denote the angle of incidence with q measured on the plane of incidence.

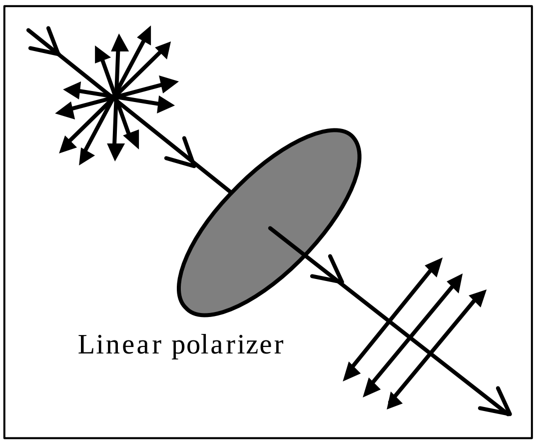


Figure 3: Transmission of a lightwave through a linear polarizer

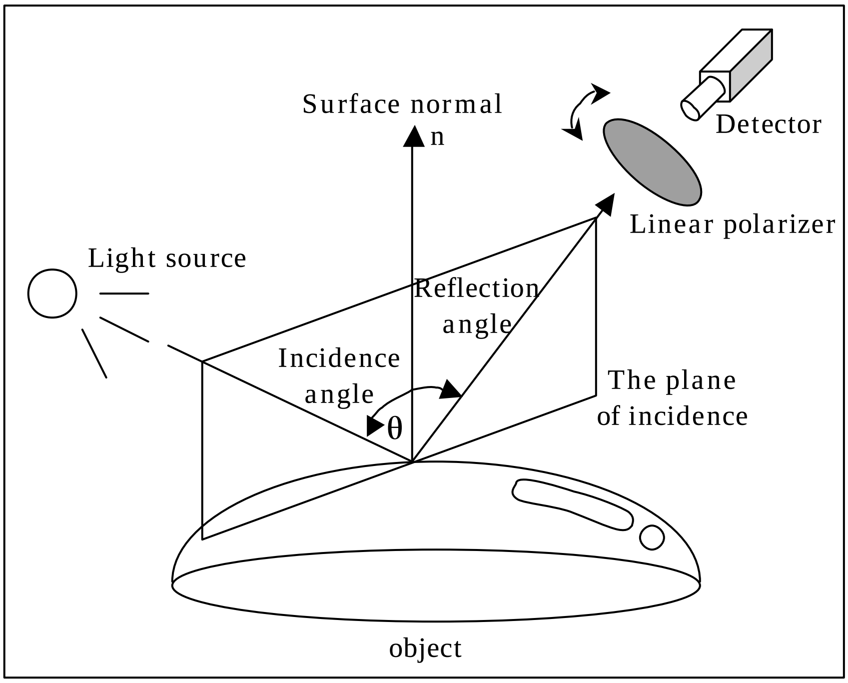


Figure 4: Surface normal of object

As seen in equation (6), intensity reflectance depends on a direction of a plane of oscillation, parallel or perpendicular. The linear combination of $I_{\max}$ and $I_{\min}$ is equal to the total light intensity of the surface component I_s.

$$I_{\max} = \frac{F_{\perp}}{F_{\|} + F_{\perp}} I_s, \quad I_{\min} = \frac{F_{\|}}{F_{\|} + F_{\perp}} I_s \tag{9}$$

Since $I_{\min}$ is the component parallel to the plane of incidence, the orientation of the plane of incidence f can be determined when $I_{\min}$ appears while rotating the polarizer.

Substituting equations (6) (9) for (8), and considering the Snell's law, the degree of polarization r is given by

$$r = \frac{2\sin q \tan q \sqrt{n^2 - \sin^2 q}}{n^2 - \sin^2 q + \sin^2 q \tan^2 q} \tag{10}$$

The degree of polarization r is a function of the angle of incidence q under a given refractive index n. Thus, from the measured degree of polarization, we can obtain the angle of incidence q from equation (10).

3 Method

3.1 Experimental Method

We propose the solution for the problem of choosing the correct angle by rotating the object. The rotating direction can be any angle but in this paper we assume that we should rotate the object from y-axis direction to z-axis direction. We rotate the object very little, for example in the simulation in section 4.1 we rotated 1 degree.

The experimental apparatus is depicted in Figure 5. An optical diffuser of a white translucent plastic sphere whose diameter is 40 cm is used to lighten the object from all the direction. The diffuser is illuminated using three incandescent electric lamps placed at intervals of 120 degrees. This makes a spherical extended light source, which will contribute to detect the highlight of the whole surface of the object. An object is placed in the center of this sphere. Using a CCD camera, images of the object are taken through a hole located at the north pole of the sphere.

We rotated the polarizer from 0 degree to 180 degrees in every 5 degrees. Since the sampling point is not continuous, we matched these sampling point to sine curve with using least squares method and detected the maximum and minimum of the light intensity. We get the polarization using maximum and minimum of the light intensity from the equation (8). After measuring the object, we rotate the object and measure it once more and we again get the polarization. At the same time, the angle of the polarizer at the minimum intensity gives the orientation on the x-y plane. The actual experiment produces four matrixes, which are polarization matrix and x-y orientation matrix both in non-rotated and rotated object. Note that the both incidence angle and x-y orientation has two angles each. The two incidence angles will be produced from the equation (10) using the polarization. The x-y orientation angle will be produced from the angle of the polarizer at the minimum intensity $I_{\min}$, since from equation (9), the parallel component to the plane of incidence is $I_{\min}$. In thinking over the value of a certain angle of the polarizer, say a, we can't distinguish the angle a from the angle $a+180°$. So, also the two x-y orientation angles will be produced and the difference between the two angles is 180°.

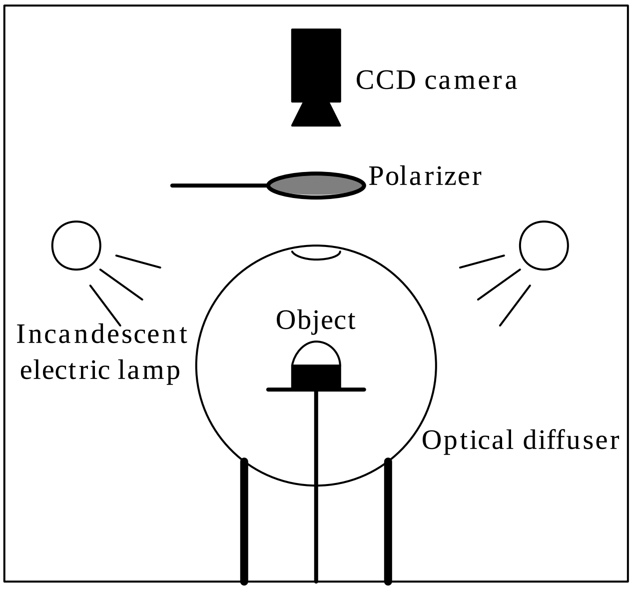


Figure 5: Experimental setup

3.2 Algorithm

The graph of equation (10) is shown in Figure 6. As you can see, if we give a polarization r it produces two incidence-angle q_1, q_2. To measure the object automatically, we should choose the correct angle q automatically. We propose that the very little rotation of the object will be the solution of this problem. First, we measure the object and next, rotate the object very little and measure the rotated object. This produces two matrixes of polarization. The algorithm chooses the correct angle automatically by comparing these two matrixes. We paid attention to the Brewster angle since the polarization of Brewster angle is 1 and the angle will be calculated only one angle. We call the contour line of the Brewster angle as Brewster line – the line where we connect the point of the Brewster angle – and we call the area enclosed by two Brewster lines as Brewster area. We detect the Brewster line and Brewster area at each matrix. This algorithm requires the object's surface to be continuous and smooth – able to calculate first derivation.

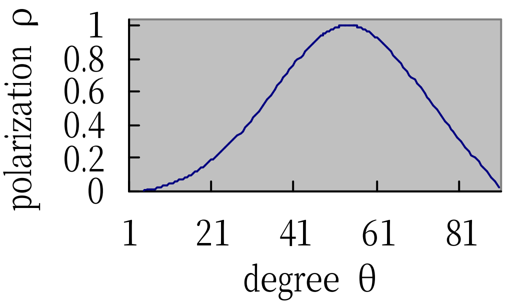


Figure 6: degree-polarization graph

There are only six patterns of surface shapes, since the Brewster area angle has only two patterns, larger than the boundary of the area (i.e. Brewster angle) or smaller than the boundary of the area. Figure 7 shows the six patterns of surface shapes between two Brewster points. In pattern 1, 3, 4, and 6, area angle is smaller than the boundary and in pattern 2, 5, area angle are larger than the boundary. In pattern 1, 2, area angle is pointing left, in pattern 5, 6, area angle is pointing right, and in pattern 3, 4, area angle changes left to right (right to left) at the top (bottom) of the mountain (valley).

If we rotate the objects they changes as Figure 7. We can detect the surface shape by looking at the changes of the value of the minimum polarization between the two Brewster lines. Angle will be larger through the rotation in pattern 1 and 2 and smaller in pattern 5 and 6. In pattern 3 and 4, angle will not change through the rotation. In pattern 1, angle is near to 0 degree but after rotation, angle goes near to

Brewster angle, so polarization increases through rotation. In pattern 2, angle is near to Brewster angle but after rotation, angle goes near to 90 degrees, so polarization decreases through rotation. In pattern 3 and 4, angle neither increase nor decrease. In pattern 5, angle is near to 90 degrees but after rotation, angle goes near to Brewster angle, so polarization increases through rotation. In pattern 6, angle is near to Brewster angle but after rotation, angle goes near to 0 degree, so polarization decreases through rotation.

The x-y orientation angle, which also has two angles, can also be chosen. Notice that x-y orientation angle changes only at the top (bottom) in pattern 3 (and 4). The x-y orientation angle will turn left to right in pattern 3 and right to left in pattern 4. We can assume that the left most edge is heading left so we just scan left to right to choose the correct angle. Start with heading left, and if we reached at the top or bottom, we switch the angle to left or right.

In determining the area between the object edge and the first Brewster line from the object edge, the x-y orientation angle is equal to that of the area between the first Brewster line and the second Brewster line. And the incident angle is exactly the opposite to that of the area between the first Brewster line and the second Brewster line.

You will have a question how we detected the Brewster line. We choose one line parallel to y-axis including top (or bottom) of the object and analyze the line to detect the point of the Brewster angle. The polarization matrix is given by the experiment, so from analyzing the 1-dimensional array, we can get the Brewster point. We choose several points from the array (for example, in the simulation we choose 16 points) and applied least squares method of quadratic function to find out the top of the function. If the top point is included in the chosen points, we can determine the maximum of the chosen points as the Brewster point. We use this algorithm to detect Brewster point of this line. In using this algorithm, we also choose the point which is not the Brewster angle even if it is a top of the quadratic function. But it is not an unwanted thing but an appreciated thing. The top of the quadratic function of polarization is a special point, which we also have to regard as a Brewster point. And in the actual experiment, we can't detect the polarization of Brewster angle as the value 1, instead of the value less than 1, and also for this reason, regarding the top of the function as the Brewster angle is necessary. We just choose one line of the whole polarization matrix. And the Brewster line of the other part of the polarization matrix can be determined by analyzing the point near to the above-mentioned line or already determined point, considering that the point is in its area if the polarization of the point is less than the boundary polarization.

We can summarize the whole measurement algorithm as follows:

1. We setup the object on the experimental apparatus.
2. While rotating the polarizer, we measure the light intensity to find the $I_{\max}$ and $I_{\min}$ at each pixel.
3. By finding the polarizer rotation angle that provides the minimum intensity, we obtain the two x-y orientation angles f_1, f_2.
4. The degree of polarization is given by equation (8) with measured $I_{\max}$ and $I_{\min}$.
5. Equation (10) provides the degree of polarization from the refractive index n and the incidence angle q. By inversely solving the equation from a given refractive index n and measured degree of polarization r, we can obtain the two incidence angles q_1 and q_2.
6. We rotate the object from y-axis to z-axis very little and do the same thing as 1-4.
7. We detect the Brewster angle of each two matrixes of polarization.
8. We compare the two matrixes and consider the change of the angle of the point of minimum polarization between two Brewster lines and choose the correct angle. We produce the surface orientations.

Figure 7: Changes of the surface through rotating the object

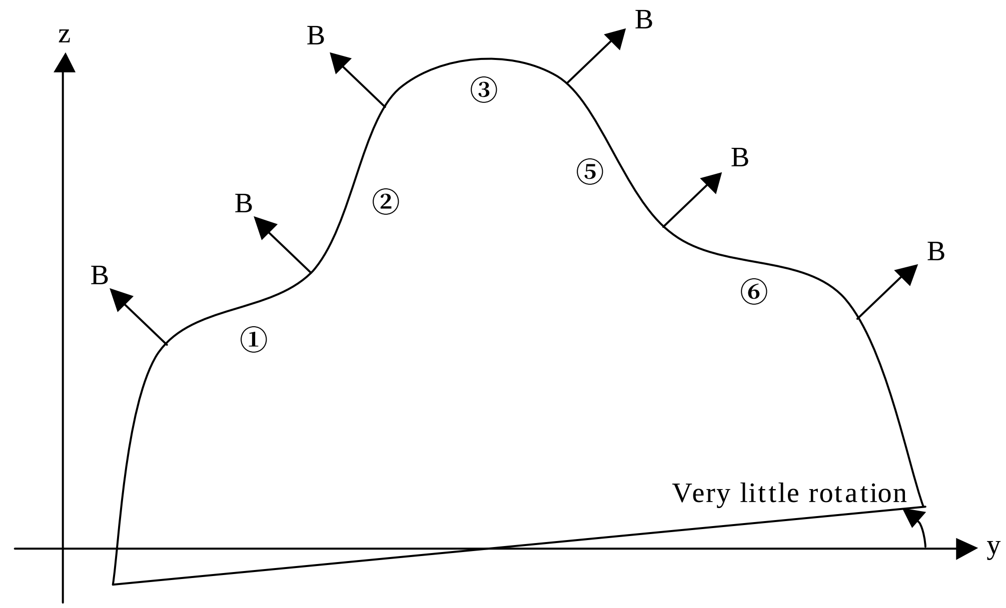


Figure 7-1: How to rotate the object (“B” represents Brewster angle)

①−⑥: Patterns

Pattern	Orientation of left Brewster angle	Internal angles compared with Brewster angle	Orientation of right Brewster angle	Minimum-polarization angle changes	Polarization changes
①	←	S	←	0→B	+
②	←	L	←	B→90	−
③	←	S	→	0	0
④	→	S	←	0	0
⑤	→	L	→	90→B	+
⑥	→	S	→	B→0	−

Table Figure 7-2: Summary of Figure 7 (Figure 7-3 to Figure 7-7)

B: Brewster angle

L, S: Larger, Smaller

Following: “B” represents Brewster angle,

“min” represents the angle of the point of minimum polarization

(minimum-polarization angle) between the two Brewster angles.

Following picture shows the all patterns of surface between the two Brewster angles,

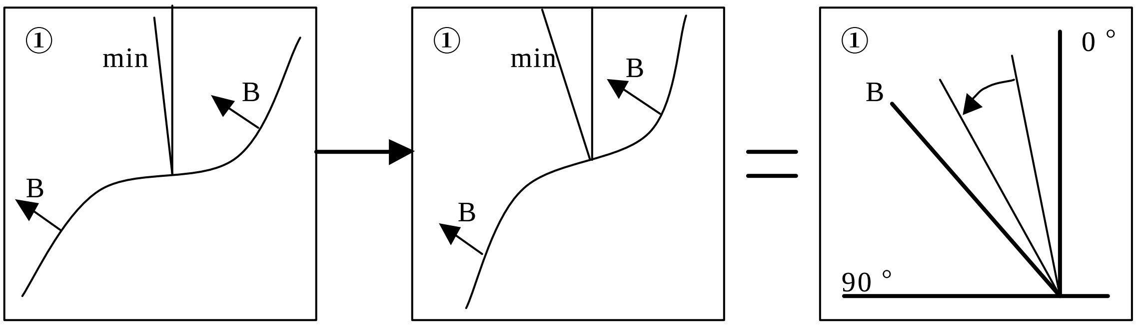


Figure 7-3: Pattern 1

Angle between the two Brewster angles is smaller than those Brewster angles

Minimum-polarization angle changes from near 0 degree to near Brewster angle

Polarization increases

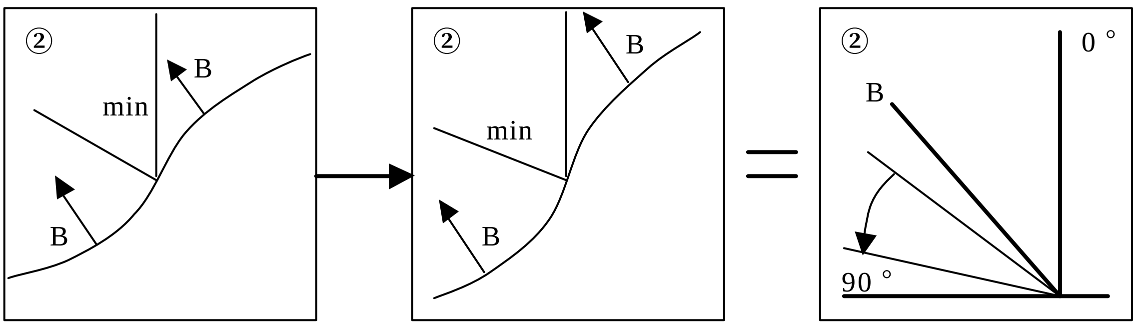


Figure 7-4: Pattern 2

Angle between the two Brewster angles is bigger than those Brewster angles

Minimum-polarization angle changes from near Brewster angle to near 90 degrees

Polarization decreases

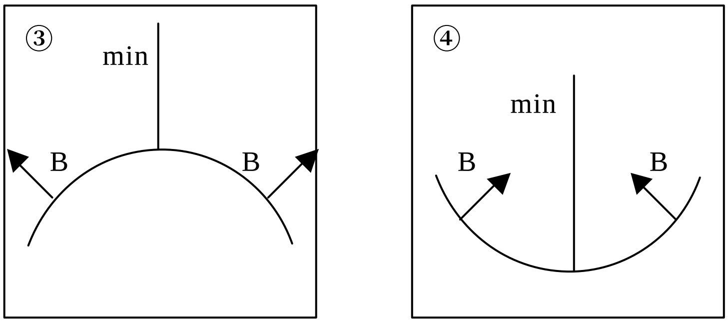


Figure 7-5: Pattern 3, 4

Angle between the two Brewster angles is smaller than those Brewster angles

Minimum-polarization angle is 0 degree and doesn’t changes

Polarization doesn’t changes

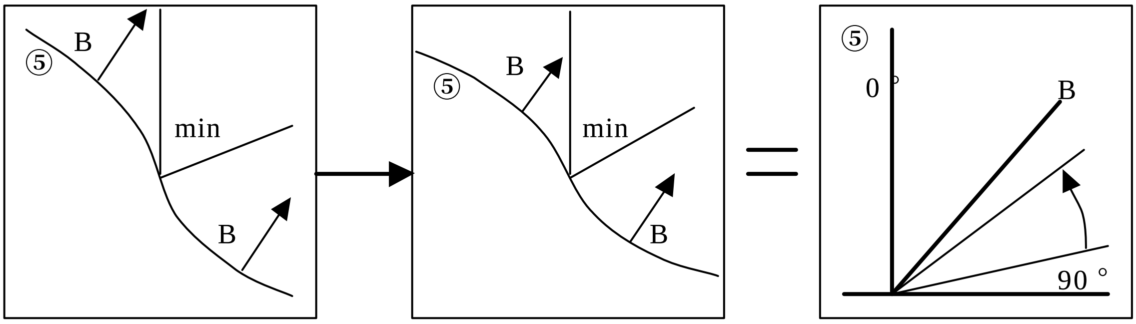


Figure 7-6: Pattern 5

Angle between the two Brewster angles is bigger than those Brewster angles

Minimum-polarization angle changes from near 90 degrees to near Brewster angle

Polarization increases

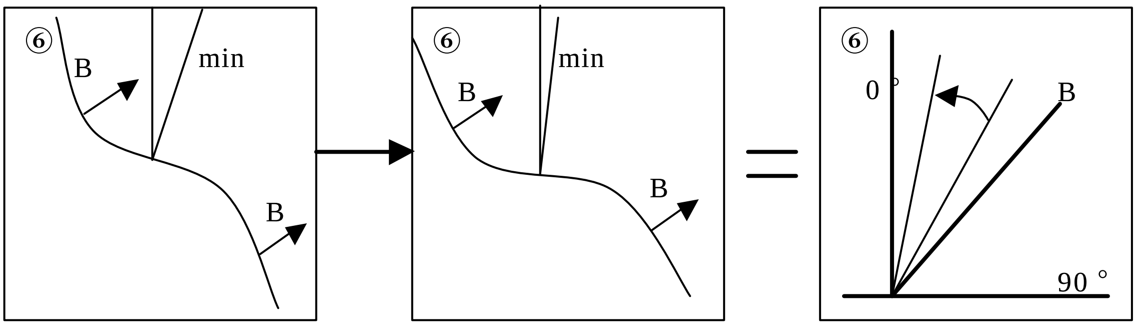


Figure 7-7: Pattern 6

Angle between the two Brewster angles is smaller than those Brewster angles

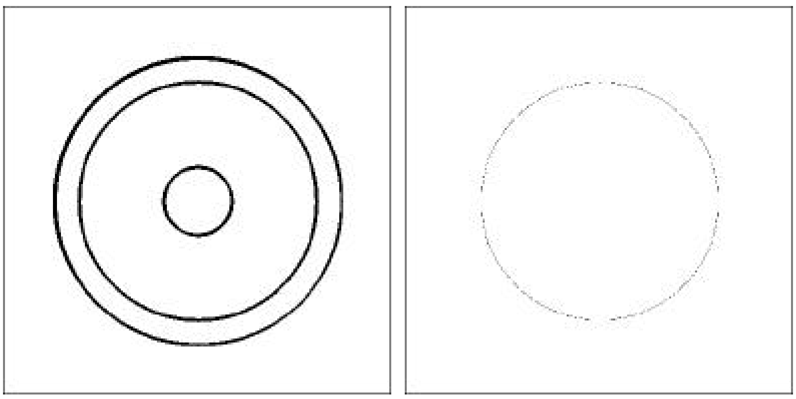
Minimum-polarization angle changes from near Brewster angle to near 0 degree

Polarization decreases

4 Experiment

4.1 Simulation

In simulation we rotated the object 1-degree, used 400x400 pixel, and set refractive index as 1.3. We simulated on 3 shapes as shown in Figure 9, Figure 10, and Figure 11. Each figure shows the original simulation shape, result shape, cross section of the original simulation shape, cross section of the rotated original simulation shape, polarization of the original simulation shape (cross section), polarization of the rotated original simulation shape (cross section), and cross section of the result shape. Figure 9 is a shape like a corn with one shoulder, Figure 10 is a shape like a doughnut and Figure 11 is the doughnut with a corn in the middle of it. As you can see, this algorithm is successful with choosing the correct angle. The result shape is almost same as the original shape. At the same time, you can also see that these result shapes are a little different to the original ones, such as the decrease of the height or the sharpness of the top of the object. This is not caused by the angle-choosing algorithm but by the simple smoothing algorithm we used. If we improve the algorithm of smoothing, the result shape will be more similar to the original shape. Setting the result shape aside, with regard to the correctness of choosing the correct angle of four angles, there is very little error. Figure 8 shows the choosing error. There is error only at the Brewster angle. Since the two incident angles are almost same near Brewster angle, the error is small enough to say that this algorithm is very useful.

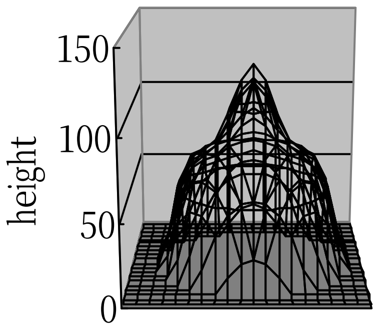


Left: Edge line and Brewster line

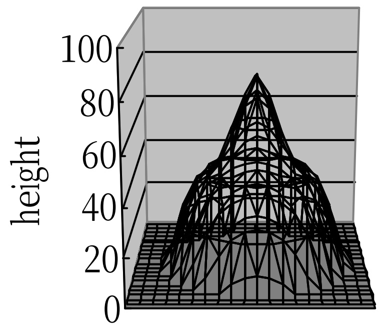
Right: The point where the algorithm choosed the wrong angle

Note: This object is of “Figure 9: Simulation result 1”.

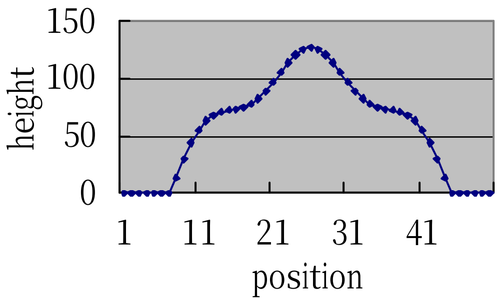
Figure 8: Error of choosing angle and Brewster line



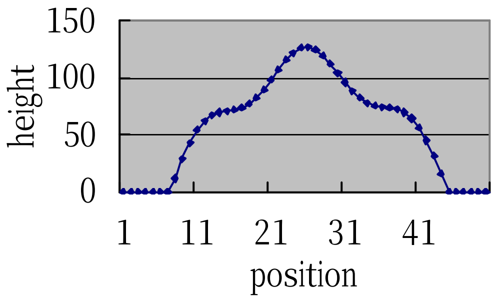
Original simulation shape



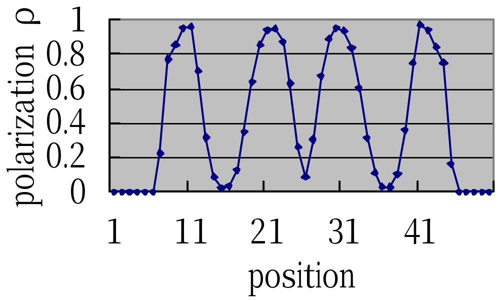
Result shape



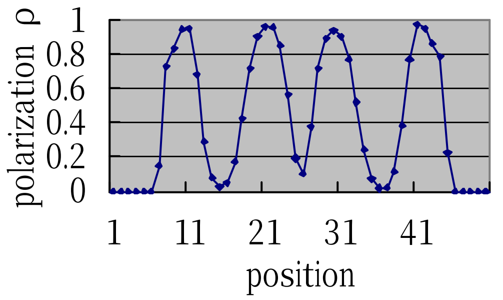
Original simulation shape



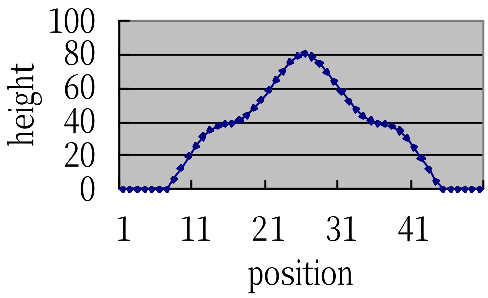
Original simulation shape (rotated)



Polarization (original shape)

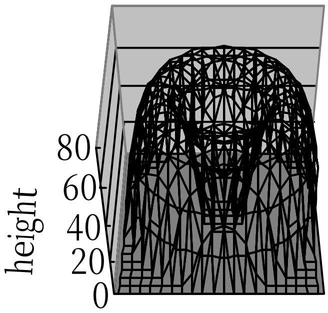


Polarization (original shape) (rotated)

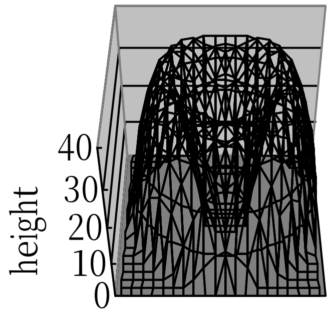


Result shape

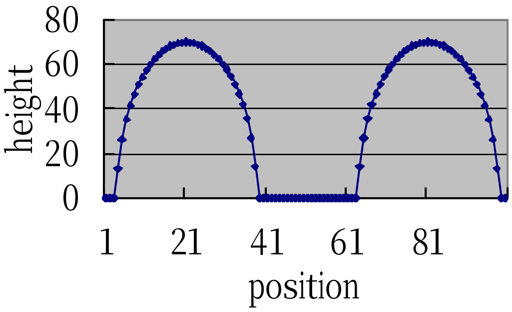
Figure 9: Simulation result 1



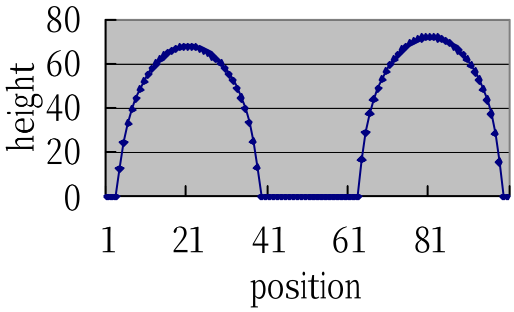
Original simulation shape



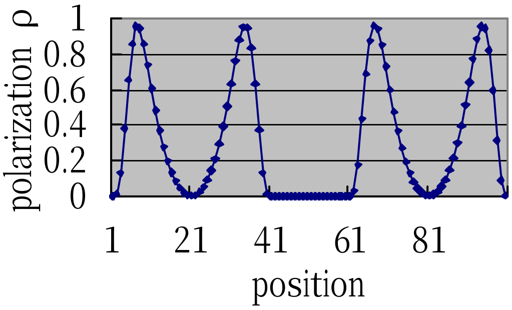
Result shape



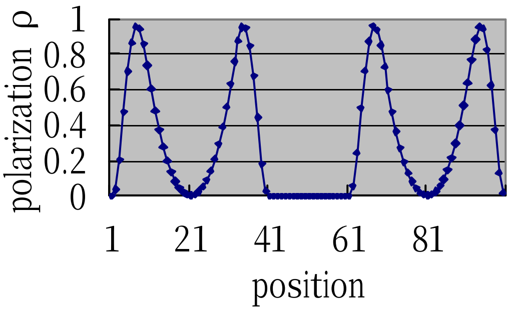
Original simulation shape



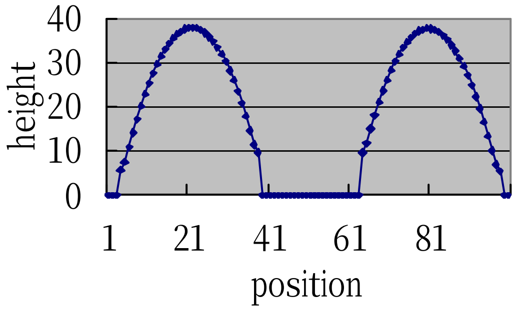
Original simulation shape (rotated)



Polarization (original shape)

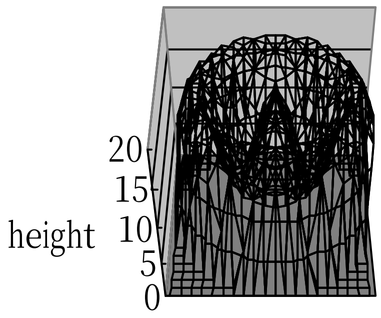


Polarization (original shape) (rotated)

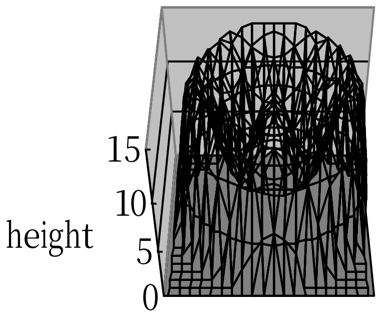


Result shape

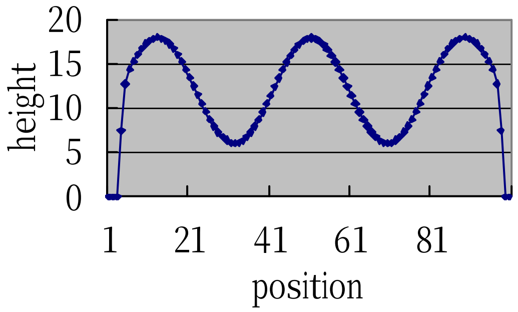
Figure 10: Simulation result 2



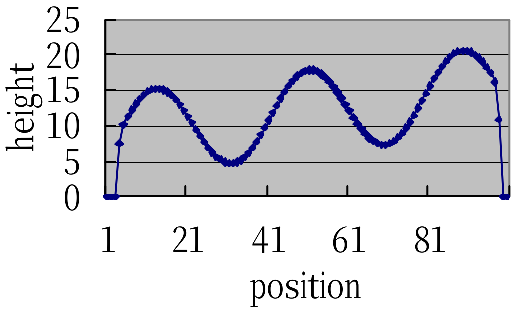
Original simulation shape



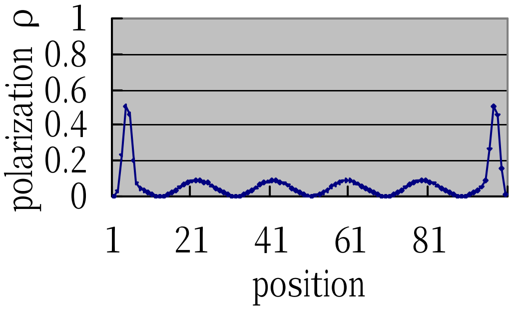
Result shape



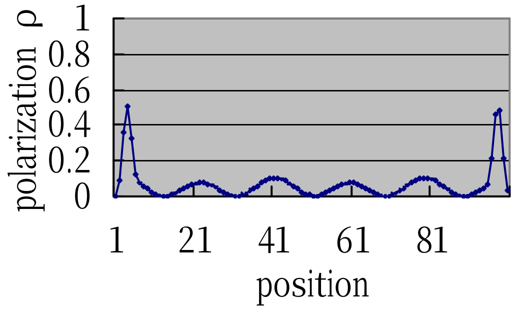
Original simulation shape



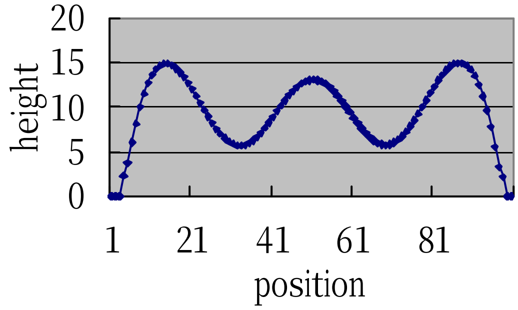
Original simulation shape (rotated)



Polarization (original shape)



Polarization (original shape) (rotated)



Result shape

Figure 11: Simulation result 3

5 Conclusion

This paper introduced a method for measuring surface orientations of transparent objects using polarization in highlight. The most important: we proposed a method for choosing the correct angle of the surface orientations automatically. Though the actual experiment can measure the polarization and angle, it produces four candidates of the angle. We rotated the object very little and detected the Brewster line and from comparison between the non-rotated and rotated object data, we find the correct angle by looking at the change of the polarization through the rotation. The results show that this method is effective to choose the correct angle.

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